Simple Harmonic Motion (SHM)

Basics, Differential Equation and Its Solution

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1. Introduction

In Physics, many systems execute **to-and-fro motion** about a fixed position. Such motions are called **oscillatory motions**. Among them, **Simple Harmonic Motion (SHM)** is the most fundamental and ideal form of oscillatory motion. It provides the basis for understanding vibrations, waves, acoustics, and many phenomena in classical and modern physics.

2. Oscillatory and Periodic Motion

- **Oscillatory motion**: Motion in which a particle moves repeatedly about a mean position.
- **Periodic motion**: Motion that repeats itself after equal intervals of time.

Simple harmonic motion is both **oscillatory and periodic**, but all periodic motions are not necessarily SHM.

3. Basic Concept of Simple Harmonic Motion

A particle is said to execute **simple harmonic motion** if the force acting on it satisfies the following conditions:

1. The force is always directed towards a fixed mean position.
2. The magnitude of the force is directly proportional to the displacement from the mean position.

4. Definition of Simple Harmonic Motion

Simple Harmonic Motion (SHM) is defined as:

> The motion in which the restoring force acting on a particle is directly proportional to its displacement from the mean position and always directed towards that position.

Mathematically,

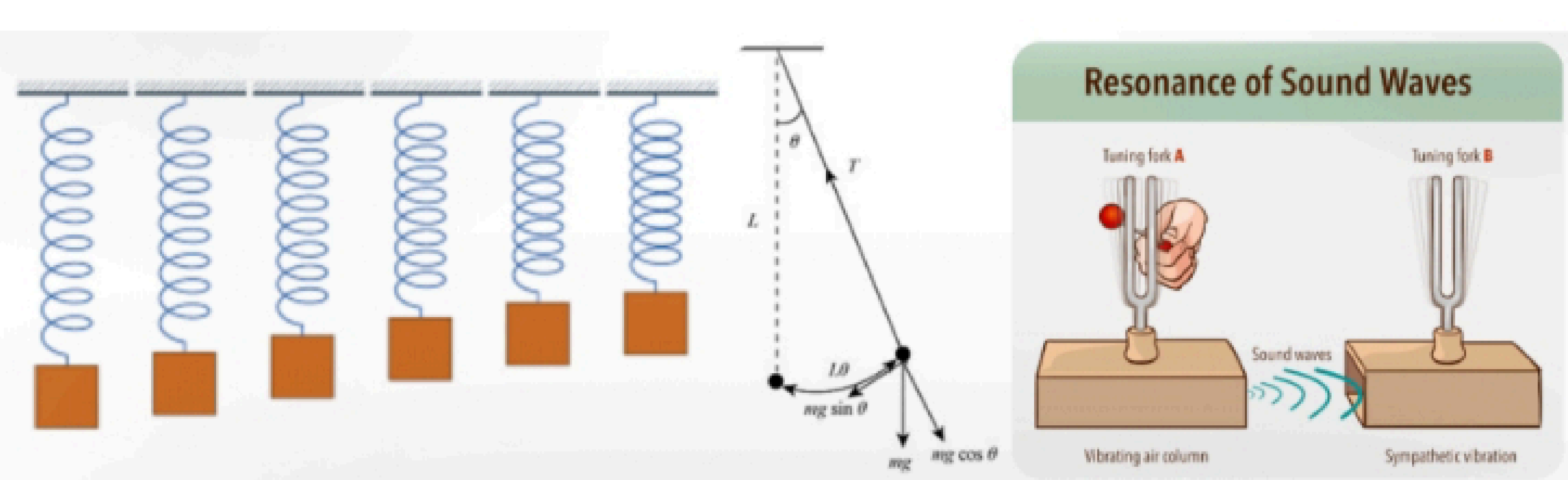
$$F \propto -x$$

$$F = -kx$$

where
x = displacement from mean position
k = force constant
Negative sign indicates restoring nature of the force.

5. Examples of SHM

- Motion of a mass attached to a spring
- Small oscillations of a simple pendulum
- Vibrations of tuning fork
- Oscillation of atoms in a crystal lattice



Simple Harmonic Motion

According to **Newton's second law of motion:**

$$F = ma = m\frac{d^2x}{dt^2}$$

For SHM, restoring force:

$$F = -kx$$

Equating,

$$m\frac{d^2x}{dt^2} = -kx$$

Rearranging,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Let,

$$\omega^2 = \frac{k}{m}$$

Hence, the **differential equation of SHM** is:

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

This is a **second-order linear differential equation.**

7. Physical Meaning of the Differential Equation

From the equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

It is clear that:

- Acceleration is proportional to displacement.
- Acceleration is opposite in direction to displacement.

This condition is the **characteristic property of simple harmonic motion.**

8. Solution of the Differential Equation of SHM

Consider the differential equation:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Assume a solution of the form:

$$x = A\sin(\omega t + \phi)$$

First derivative:

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

Second derivative:

$$\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

Substituting into the differential equation:

$$-A\omega^2\sin(\omega t + \phi) + \omega^2 A\sin(\omega t + \phi) = 0$$

Hence, the assumed function satisfies the differential equation.

9. General Solution of SHM

Therefore, the **solution of the differential equation of SHM** is:

$$\boxed{x = A\sin(\omega t + \phi)}$$

$$\boxed{x = A\cos(\omega t + \phi)}$$

where
A = amplitude of motion
ω = angular frequency
ϕ = phase constant

10. Time Period and Frequency

The angular frequency is related to time period as:

$\omega = \frac{2\pi}{T}$

Hence,

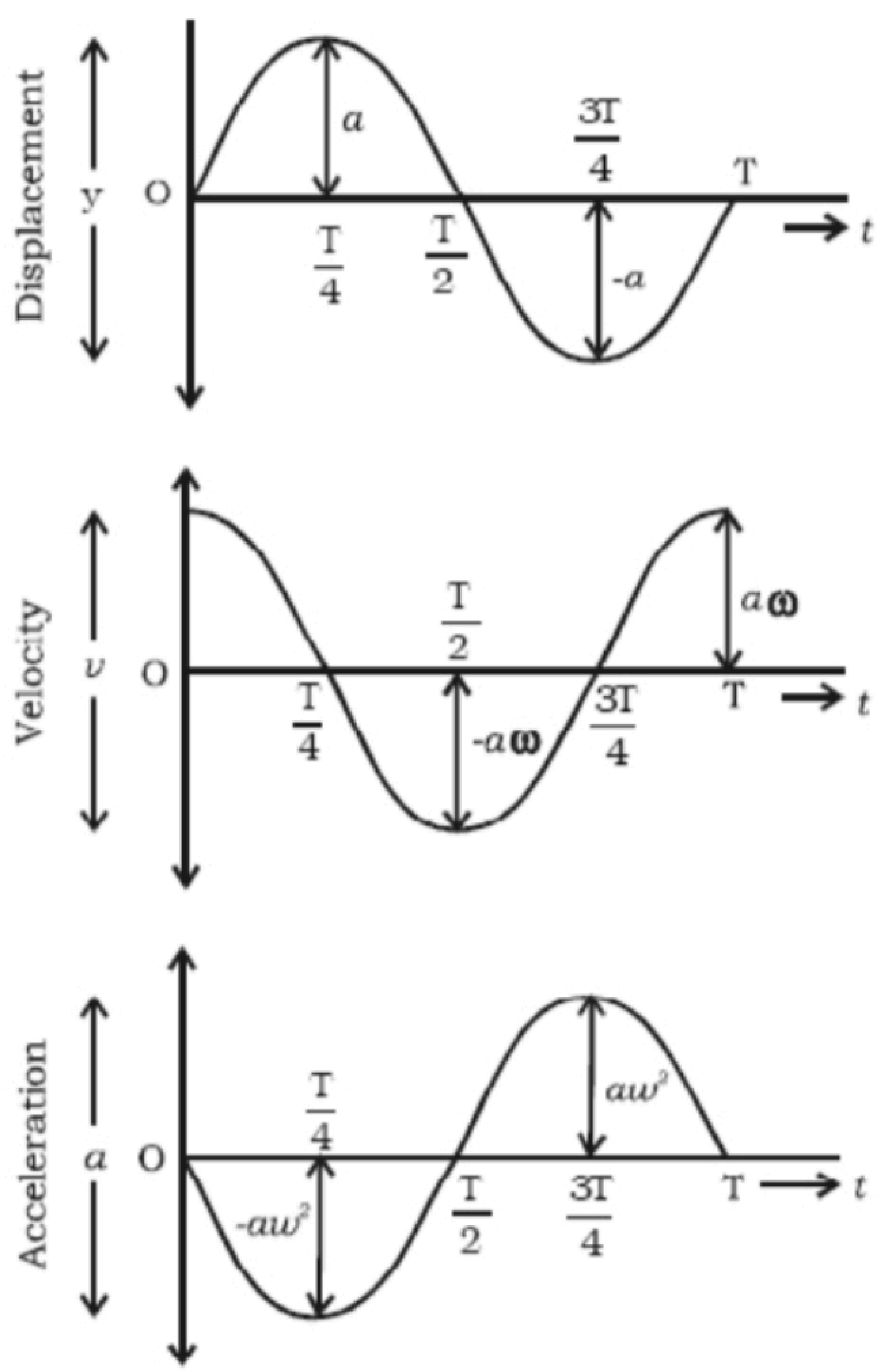
$$\boxed{T = \frac{2\pi}{\omega}}$$

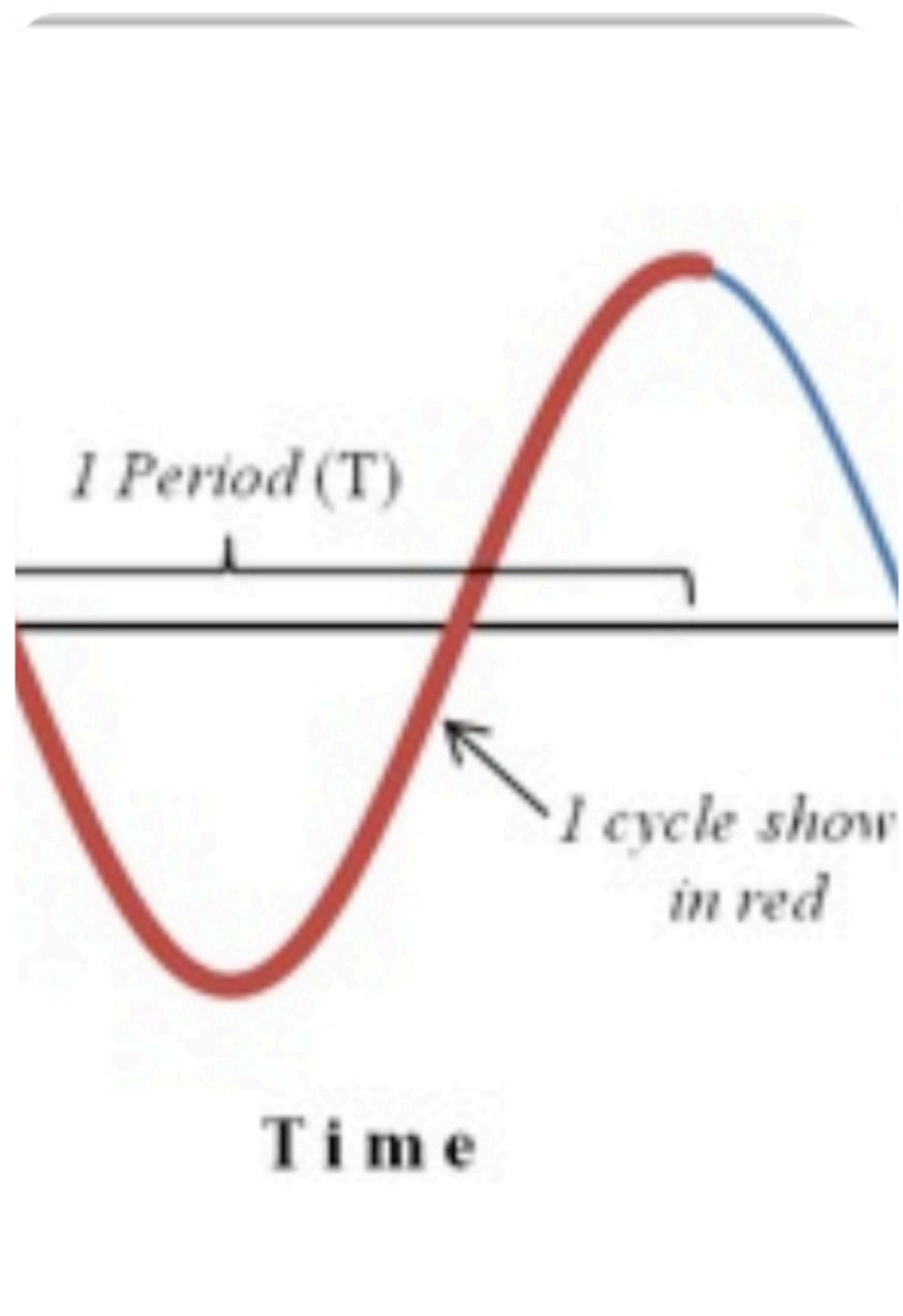
Frequency:

$$\boxed{f = \frac{1}{T}}$$

11. Graphical Representation of SHM

The displacement–time graph of SHM is a **sinusoidal curve**.





12. Importance of SHM

- Foundation of wave motion
- Important in acoustics, optics, and mechanics
- Basis of oscillations in electrical circuits
- Useful in understanding atomic and molecular vibrations

13. Conclusion

Simple harmonic motion is the most fundamental type of oscillatory motion in Physics. Its differential equation mathematically represents the restoring nature of force, and its sinusoidal solution completely describes the motion of the particle. A clear understanding of SHM is essential for the study of vibrations, waves, and advanced topics in Physics.